

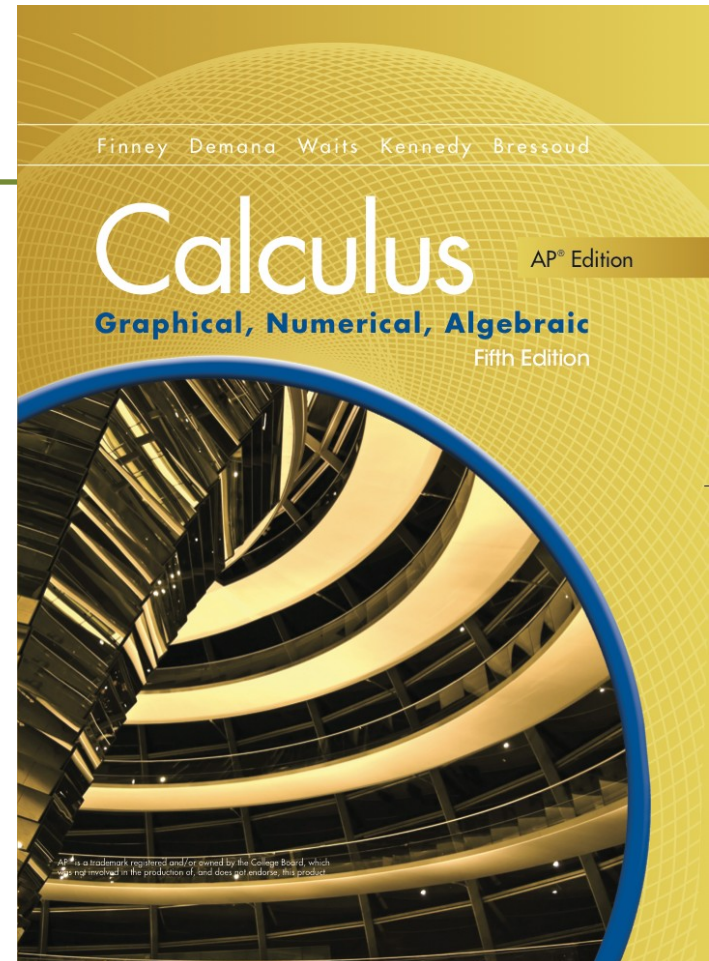
Chapter 7

Differential Equations and Mathematical Modeling



Section 7.3

Antidifferentiation by Parts



Quick Review

1. Find $\frac{dy}{dx}$ given $y = \tan^{-1} 3x$
2. Solve for x in terms of y given $y = \tan^{-1} 3x$
3. Solve the differential equation $\frac{dy}{dx} = e^{3x}$.
4. Solve the initial value problem $\frac{dy}{dx} = x + \cos x$, $y(0) = 3$.

Quick Review Solutions

1. Find $\frac{dy}{dx}$ given $y = \tan^{-1} 3x$ $\frac{dy}{dx} = \frac{3}{1+9x^2}$
2. Solve for x in terms of y given $y = \tan^{-1} 3x$ $x = \frac{1}{3} \tan y$
3. Solve the differential equation $\frac{dy}{dx} = e^{3x}$. $y = \frac{1}{3} e^{3x} + C$
4. Solve the initial value problem $\frac{dy}{dx} = x + \cos x$, $y(0) = 3$.
 $y = \frac{1}{2} x^2 + \sin x + 3$

What you'll learn about

- Deriving the parts formula from the Product Rule
- Repeated application of the parts formula
- Solving for the unknown integral
- Tabular integration (for repeated application of the parts formula)
- Antiderivatives of logarithmic and inverse trigonometric functions

... and why

The Product Rule relates to derivatives as the technique of parts relates to antiderivatives.

Integration by Parts Formula

$$\int u dv = uv - \int v du$$

Example Using Integration by Parts

Evaluate $\int x \sin x dx$

Use $\int u dv = uv - \int v du$ with

$$u = x \quad dv = \sin x dx$$

$$du = dx \quad v = -\cos x$$

$$\begin{aligned}\int x \sin x dx &= -x \cos x + \int \cos x dx \\ &= -x \cos x + \sin x + C\end{aligned}$$

Example Repeated Use of Integration by Parts

Evaluate $\int 2x^2 e^x dx$

$$\text{Let } u = 2x^2 \quad dv = e^x dx$$

$$du = 4x dx \quad v = e^x$$

$$\int 2x^2 e^x dx = 2x^2 e^x - 4 \int x e^x dx$$

$$\text{Now let } u = x \quad dv = e^x dx$$

$$du = dx \quad v = e^x$$

$$\begin{aligned} \int 2x^2 e^x dx &= 2x^2 e^x - 4 \left[x e^x - \int e^x dx \right] \\ &= 2x^2 e^x - 4x e^x + 4e^x + C \end{aligned}$$

Example Solving for the Unknown Integral

Evaluate $\int e^x \sin x dx$

Let $u = e^x$ $dv = \sin x dx$

$$du = e^x dx \quad v = -\cos x$$

$$\int e^x \sin x dx = -e^x \cos x + \int e^x \cos x dx$$

Now let $u = e^x$ $dv = \cos x dx$

$$du = e^x dx \quad v = \sin x$$

$$\int e^x \sin x dx = -e^x \cos x + e^x \sin x - \int e^x \sin x dx$$

$$2 \int e^x \sin x dx = -e^x \cos x + e^x \sin x$$

$$\int e^x \sin x dx = \frac{-e^x \cos x + e^x \sin x}{2} + C$$

Example Antidifferentiating $\ln x$

Find $\int \ln x dx$

Let $u = \ln x$ $dv = dx$

$$du = \frac{1}{x} dx \quad v = x$$

$$\int \ln x dx = x \ln x - \int (x) \left(\frac{1}{x} \right) dx$$

$$= x \ln x - \int dx$$

$$= x \ln x - x + C$$

Quick Quiz Sections 7.1 – 7.3

Solve the following problems without using a graphing calculator.

1. Which of the following differential equations would produce the slope field shown below?

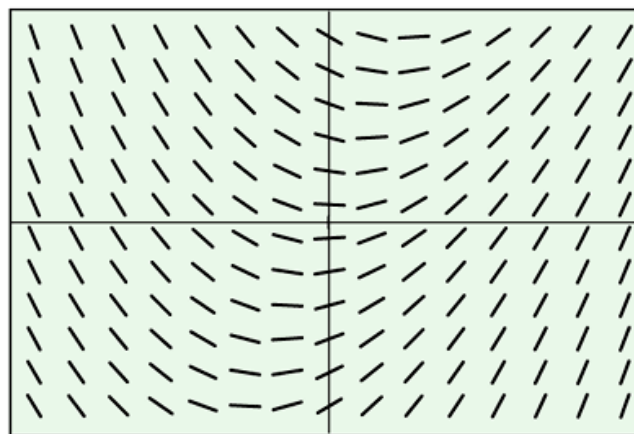
(A) $\frac{dy}{dx} = y - 3x$

(B) $\frac{dy}{dx} = y - \frac{x}{3}$

(C) $\frac{dy}{dx} = y + \frac{x}{3}$

(D) $\frac{dy}{dx} = y + \frac{x}{3}$

(E) $\frac{dy}{dx} = x - \frac{x}{3}$



Quick Quiz Sections 7.1 – 7.3

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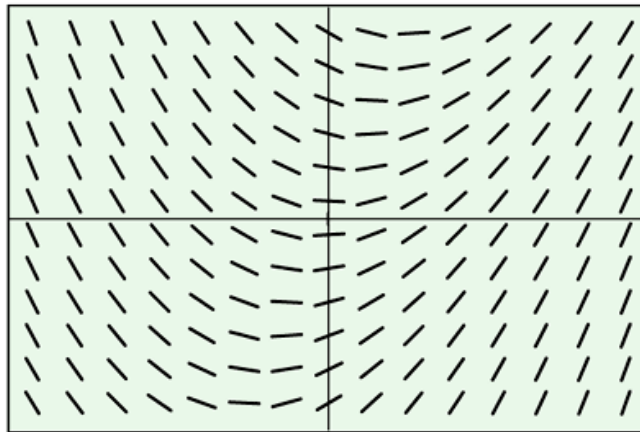
(A) $\frac{dy}{dx} = y - 3x$

(B) $\frac{dy}{dx} = y - \frac{x}{3}$

(C) $\frac{dy}{dx} = y + \frac{x}{3}$

(D) $\frac{dy}{dx} = y + \frac{x}{3}$

(E) $\frac{dy}{dx} = x - \frac{x}{3}$



Quick Quiz Sections 7.1 – 7.3

2. If the substitution $\sqrt{x} = \sin y$ is made in the integrand

of $\int_0^{1/2} \frac{\sqrt{x}}{\sqrt{1-x}} dx$, the resulting integral is

(A) $\int_0^{1/2} \sin^2 y dy$

(B) $2 \int_0^{1/2} \frac{\sin^2 y}{\cos y} dy$

(C) $2 \int_0^{\pi/4} \sin^2 y dy$

(D) $\int_0^{\pi/4} \sin^2 y dy$

(E) $2 \int_0^{\pi/6} \sin^2 y dy$

Quick Quiz Sections 7.1 – 7.3

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Quick Quiz Sections 7.1 – 7.3

3. $\int x e^{2x} dx =$

(A) $\frac{x e^{2x}}{2} - \frac{e^{2x}}{4} + C$

(B) $\frac{x e^{2x}}{2} - \frac{e^{2x}}{2} + C$

(C) $\frac{x e^{2x}}{2} + \frac{e^{2x}}{4} + C$

(D) $\frac{x e^{2x}}{2} + \frac{e^{2x}}{2} + C$

(E) $\frac{x e^{2x}}{4} + C$

Quick Quiz Sections 7.1 – 7.3

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